Chapter 2

Filters

Filter is the name given to a circuit whose function it is to selectively block or pass a range of signal frequencies. This is called *selectivity*. Probably the most obvious examples of filters are the tone controls in your hi-fi amplifier. What happens when you turn down the bass? The lower frequencies that arrive at the output of the amplifier are attenuated but the higher frequencies remain unchanged. This is an example of the effects of a **high pass filter**. The filter lets the higher frequencies pass but blocks the lower frequencies. In this section I'll list the basic types of filters and state some applications of filters in electronic systems.

At the completion of this chapter you should be able to:

2.1 State the purpose of filters in electrical and electronic systems.

2.2 Describe the frequency response and sketch the circuit symbols for ideal high-pass filters.

2.3 Describe the frequency response and sketch the circuit symbols for ideal low-pass filters

2.4 Describe the frequency response and sketch the circuit symbols for ideal band-pass filters

2.5 Describe the frequency response and sketch the circuit symbols for ideal band-stop filters

2.6 List the advantages of active filters over passive filters.

2.7 List the common types of practical filters, their characteristics and applications.

2.8 State and define the terms: pass band; cut-off frequency; order of filter; roll off; attenuation; pass band attenuation; bandwidth; phase shift

2.1 Purpose

In its simplest form a filter can consist of a single capacitor. When you looked at regulated power supplies you saw that a large filter capacitor was used to filter ripple from the output. And that when the frequency of the rectified waveform increased, that is, went from half-wave to full-wave, the ripple decreased. What would happen if you continued to increase the input frequency? Logically, the ripple would decrease even further. What happened to it? Let's look at Figure 2.1.

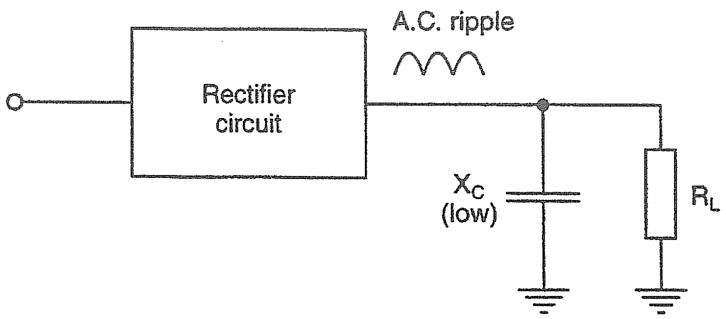


Figure 2.1 Filtering AC ripple

The ripple is AC. The capacitor has a capacitive reactance that is inversely proportional to the applied frequency:

$$\boxed{X_C = \frac{1}{2\pi f C}}$$

As the frequency increases the reactance of the capacitor decreases. To be effective the reactance of a regulated power supply filter capacitor must be very low at the ripple frequency. That is why large value capacitors are used. Don't forget – capacitive reactance is also inversely proportional to capacitance. As the frequency increases the capacitor becomes more and more like a short circuit to the AC ripple. The ripple is being passed to ground.

Example

What is the capacitive reactance of a 2200µF capacitor when the applied frequency is 50Hz?

Solution

$$X_C = \frac{1}{2\pi fC}$$

$$= \frac{1}{6.28 \times 50 \times 2200 \times 10^{-6}}$$

$$= 1.45\Omega$$

Exercise

2.1 What is the capacitive reactance of the above capacitor when the applied frequency is 100Hz?

LC filter

Obviously you can't continue to decrease the ripple by increasing the frequency of the supply; it's set at 50Hz. But you can decrease the ripple in a regulated supply by using an LC filter. An LC filter uses the reactance of an inductor in series with the reactance of the capacitor to form an AC voltage divider. See Figure 2.2.

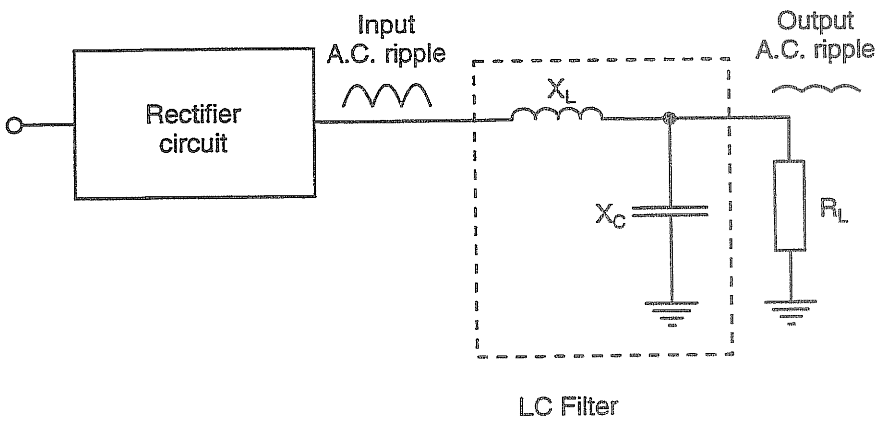


Figure 2.2 Rectifier with an LC filter

The reactance of the inductor is high at the ripple frequency:

$$X_L = 2\pi fL$$

At the same time the reactance of the capacitor is small. So, most of the input ripple to the filter is dropped across the inductor. The remaining output ripple is smoothed across the capacitor (and, hence, the load).

So, capacitors and inductors are used as filters in electrical systems to remove unwanted variations, such as ripple, from the power supply. Filters are also used in small signal applications to selectively pass and block signal frequencies.

Let's look at the various types of signal filter.

2.2 Low pass filter (LPF)

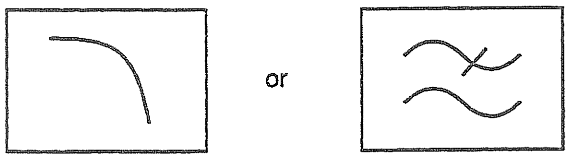


Figure 2.3 Two symbols used for low pass filters

As its name suggests (and as the symbols in Figure 2.3 show) this filter passes low frequencies but blocks high frequencies. The frequency response curve for a low-pass filter is shown in Figure 2.4.

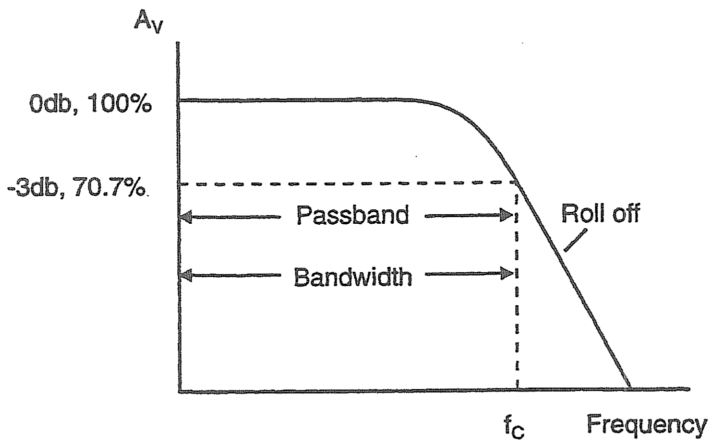


Figure 2.4 Low pass filter frequency response curve

The pass band of this filter is from DC (0Hz) up to the cut-off frequency, f_C. Therefore its bandwidth is f_C. Ideally the curve would be flat to f_C then drop instantaneously.

The rate at which the filter's frequency response curve rolls off (also called the slope of the filter) depends on the order of the filter.

Order number and poles

The order number of a filter refers to the number of reactive elements in its circuit. A first order filter has one capacitor (or one inductor) in it. A second order filter has two capacitors or two inductors or one of each, and so on.

Filters may be made up of RC circuits. An RC circuit in a filter is known as a pole. So a filter made up of a single RC circuit is referred to as a single pole filter. A two pole filter would have two RC circuits, and so on.

Since each pole contains a reactive element, a single pole filter is also a first order filter, a two pole filter is a second order filter and so on.

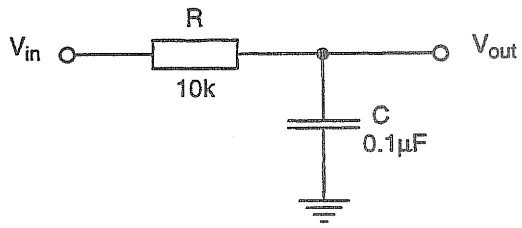


Figure 2.5 First order passive low pass filter

The figure above shows a *first order* (or *single pole*) *passive low pass filter*.

- First order or single pole means that there is only one reactive component (the capacitor) in the filter circuit.
- Passive means that there is no amplifier in the filter circuit itself.

The resistor and the capacitor form a voltage divider. The frequency of the signal applied to the filter will determine the capacitive reactance of the capacitor. This, in turn, will determine the amplitude of the voltage across the capacitor. The voltage across the capacitor is the output voltage of the filter.

As the frequency increases X_C decreases so V_{out} decreases.

At the cut off frequency the output of the filter would be 0.707 times the input voltage. In terms of decibels the relative magnitude of the output voltage with respect to the input (reference) voltage is found by:

$$dB = 20 \times \log \frac{V_1}{V_0}$$

Where: V_1 = the output voltage

V_0 = the input (reference) voltage

So, at F_C, the output in dB is found by:

$$\begin{aligned} dB &= 20 \times \log \frac{V_1}{V_0} \\ &= 20 \times \log \frac{0.707}{1} \\ &= 20 \times -0.15 \\ &= -3dB \end{aligned}$$

What is this filter's cut-off frequency? You'll need to do some vector analysis to work this out. See Figure 2.6.

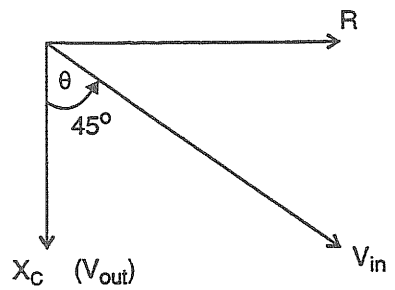


Figure 2.6 Vector analysis of the low pass filter

$V_C(V_{out})$ will be $0.707 \times V_{in}$ when θ is 45°.

This occurs when X_C = R, which in this example is 10kΩ.

You can work out the frequency at which this occurs. Since:

$$X_C = \frac{1}{2\pi f C}$$

you need $X_C = R$, so:

$$R = \frac{1}{2\pi f C}$$

Rearranging gives the cut off frequency:

$$\boxed{f_C = \frac{1}{2\pi RC}}$$

In the example:

$$f_C = \frac{1}{(6.28 \times 0.1 \times 10^{-6} \times 10 \times 10^{3})}$$

$$= 159.23\text{Hz}$$

Exercise

2.2

(a) A low pass filter uses a series resistor whose value is 3.3kΩ and a shunt capacitor that is 0.22μF. What is f_C?

(b) What value resistance would need to be used with a 0.1μF cap to achieve a cut off frequency of 600Hz?

(c) What value capacitor would need to be used with a 5.6kΩ resistor to obtain a cut off frequency of 1.1kHz?

Rates of roll off

If you increase the frequency applied to the first order filter by one decade (a tenfold increase) the, at 1592.3Hz, X_C will become one tenth. V_{out}, which is V_C, will also become (very close to) one tenth. In dB the output is now:

$$dB = 20 \times log \frac{V_1}{V_0}$$

$$= 20 \times log \frac{0.1}{1}$$

$$= 20 \times -1$$

$$= -20dB$$

Stop & Think

The roll off for a first order filter may also be specified as being 6dB per octave. An octave is a doubling of frequency. 20dB per decade and 6dB per octave are the same rate of roll off.

Let's examine the circuit in Figure 2.5.

At one octave above f_C the frequency is $2 \times 159.23 = 318.46$Hz. At this frequency X_C has halved to 5kΩ. By applying Pythagoras's Theorem to the vector diagram in Figure 2.7 we find that:

$$V_{out} = 0.447 \times V_{in}$$

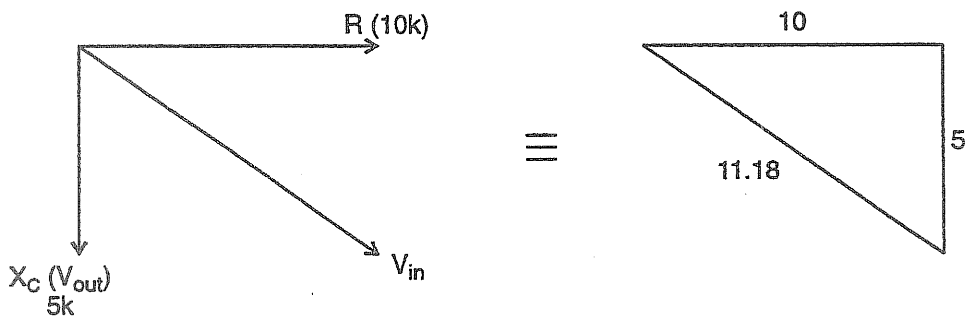


Figure 2.7 Vector diagram

In dB this is:

$$dB = 20 \times \log \frac{V_1}{V_0}$$

$$= 20 \times \log \frac{0.447}{1}$$

$$= 20 \times -.35$$

$$= -7dB \text{ **}$$

Increasing the frequency by a further octave (to 637Hz) decreases X_C to 2.5kΩ. V_{out} becomes $0.242 \times V_{in}$. This is –12.32dB. **

Increasing the frequency to 1274Hz (3 octaves above f_C) decreases X_C to 1250Ω and V_{out} falls to $0.124 \times V_{in}$. This is –18.13dB. **

Stop & Think

** These figures reflect the non-linear characteristics of the filter's slope in the early stages of roll off. However as the frequency increases the slope becomes closer to –6dB per octave. Once we are more than one decade past f_C the slope is quite linear.

Since a decade is 3.322 octaves ($2^{3.322} = 10$) we can convert dB per octave to dB per decade by multiplying the former by 3.322. So –6dB per octave × 3.322 = –20dB per decade.

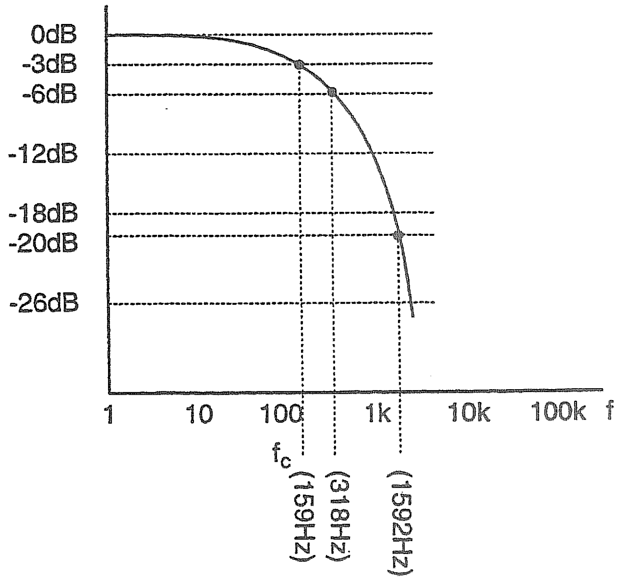


Figure 2.8 Roll off graph

If you place a second first order low pass filter after the first one (cascade them) you produce a second order filter. This introduces a further –20dB per decade roll off after its cut off frequency is exceeded. See Figure 2.9.

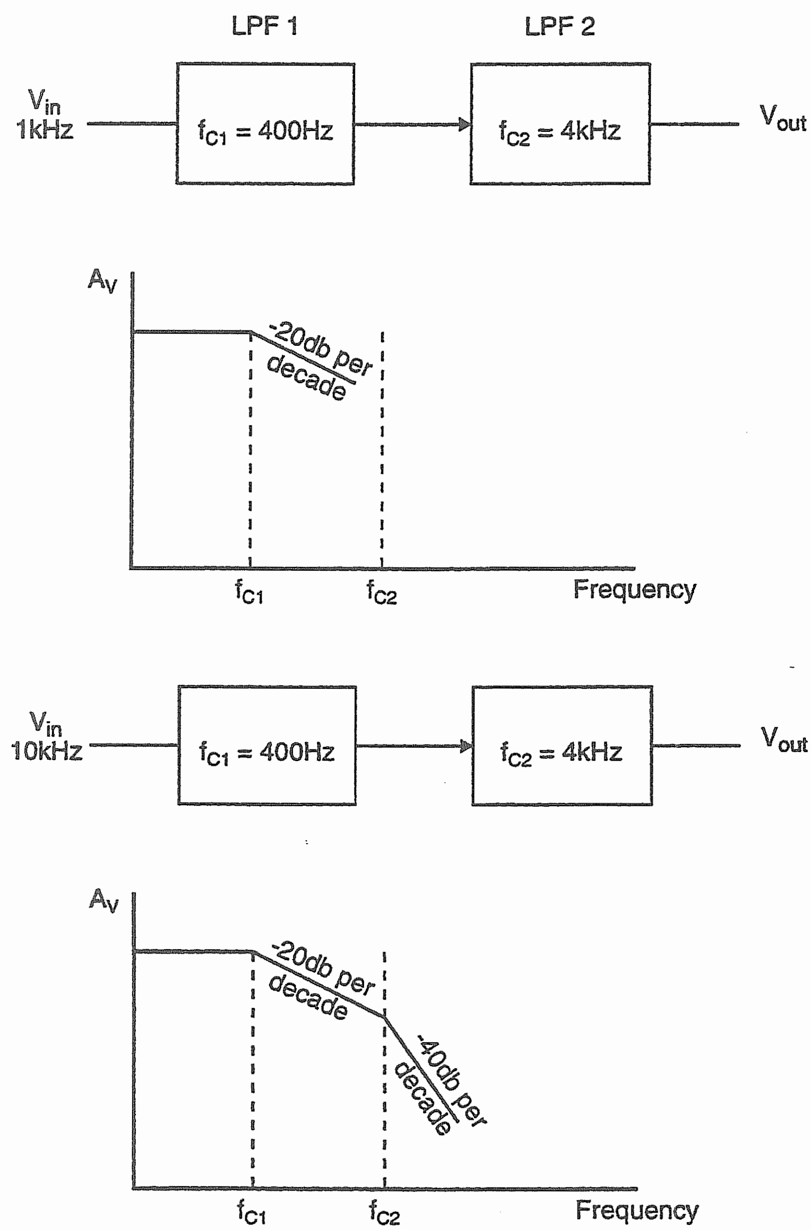


Figure 2.9 Increasing roll off by using two cascased low pass filters

Vector analysis shows that a phase shift is introduced between the input and output voltages of a filter and that this phase shift is dependent on frequency. As the order number of the filter increases this phase shift of the output also increases. The phase shifted output of the first filter provides the input to the second filter. The second filter phase shifts the signal again. (You'll be using this information later in the module when we examine Phase Shift Oscillators.The first LPF has a f_C of 400Hz. This has introduced a –20dB per decade roll off to the 1kHz input signal.

The second LPF has a f_C of 4kHz. It lets the 1kHz signal through without further attenuation.

When the input frequency is 10kHz the first LPF introduces a -20dB per decade roll off to the input voltage and so does the second LPF. The signal voltage is now rolled off at –40dB per decade, therefore the total roll off is the sum of the two filters' roll offs. If a third filter was introduced the roll off would become –60dB per decade, etc.

2.3 High pass filter (HPF)

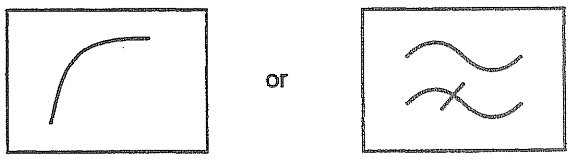


Figure 2.10 Two symbols for a high pass filter

As its name suggests (and the symbols show) this filter passes high frequencies but blocks low frequencies. The frequency response high pass curve for a filter is shown in Figure 2.11.

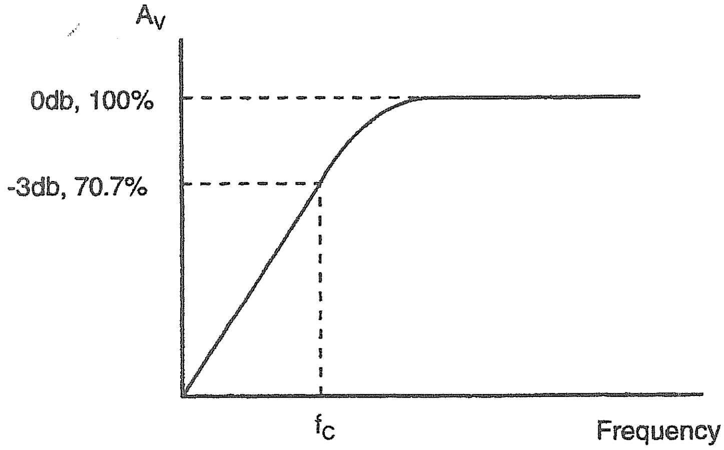


Figure 2.11 High pass filter frequency response curve

The pass band of this filter is from the cut-off frequency, f_C, up to infinity. Ideally the curve would be flat down to f_C then roll off instantaneously. However in reality the curve starts to roll off above f_C. The rate of roll off again depends on the *order* of the filter; that is, –20dB per decade per order. Once the output is 3dB down the cut-off frequency has been reached and frequencies below f_C are considered to be blocked. How is this roll off with decrease in frequency achieved?

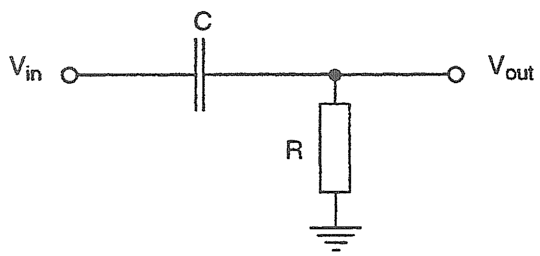


Figure 2.12 First order passive high pass filter

Figure 2.12 shows a first order passive high pass filter. Again the resistor and the capacitor form a voltage divider. The frequency of the signal applied to the filter will determine the capacitive reactance of the capacitor. This, in turn, determines the amplitude of the voltage across the capacitor.

As the frequency decreases X_C increases so V_C increases. Therefore, the voltage across the resistor (which is V_{out}) decreases. How do you find this filter's cut-off frequency? You use the same formula as before.

$$f_C = \frac{1}{2\pi RC}$$

Exercise

2.3.1 A high pass filter consists of a 8.2kΩ resistor and a 0.01μF capacitor. Draw its frequency response curve showing:

(a) Cut-off frequency

(b) Amplitude in dB of the output in the pass band of this filter

(c) Amplitude in dB of the output at the cut off frequency

(d) Amplitude in dB of the output at one tenth times the cut off frequency

2.3.2 A high pass filter has R = 6.8kΩ, C = 0.022μF. What is f_C?

Another example of high pass filtering is the coupling capacitor used between stages of an amplifier. Its effect is caused by a series capacitance and a shunt resistance. The low frequency roll off in the amplifier's frequency response is directly attributable to the value of the **coupling capacitor** and the **input impedance** of the next stage (or output load).

Exercise

2.4 A 1000μF capacitor is used to couple the output from an amplifier to an 8Ω speaker. What is the theoretical lowest possible useable frequency that the amplifier can pass on to the speaker?

2.4 Band pass filter

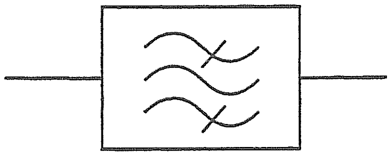


Figure 2.13 Symbol for a band pass filter

As its name suggests (and the symbol shows) this filter passes a range or band of frequencies but blocks frequencies that are above o below this band. The frequency response curve for a band-pass filte: is in Figure 2.14.

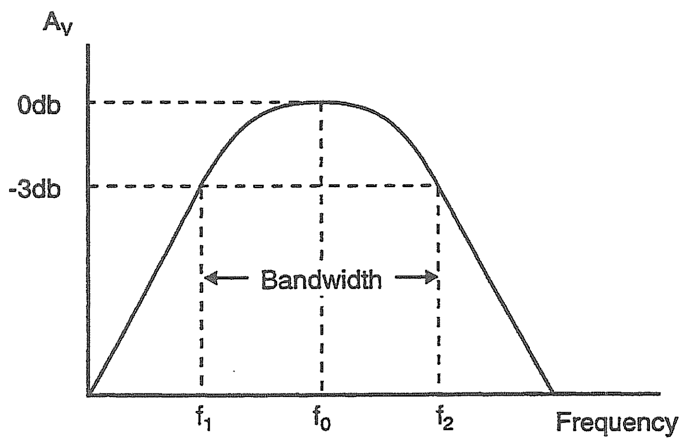


Figure 2.14 Band pass filter frequency response curve

The pass band of this filter is from the lower cut-off frequency, f_1, up to the upper cut-off frequency, f_2.

The **bandwidth** of the filter is found by:

$$\boxed{BW = f_2 - f_1}$$

Example

A band pass filter has a lower cut-off frequency of 400Hz and an upper cut-off frequency of 4kHz. What is the bandwidth of the filtei

Solution

$$BW = f_2 - f_1$$
$$= 4000 - 400$$
$$= 3600Hz$$

Centre frequency

The centre frequency, f_0, of the filter is the frequency at which the gain is the highest. As the applied frequency moves away from the centre frequency gain drops off. This frequency is the geometric mean of the cut-off frequencies and is found by:

$$\boxed{f_0 = \sqrt{(f_1 \times f_2)}}$$

Example

What is the centre frequency for the above filter?

Solution

$$f_0 = \sqrt{(f_1 \times f_2)}$$
$$= \sqrt{(400 \times 4000)}$$
$$= 1265Hz$$

So, what's the use in knowing the centre frequency?

Rearranging the last equation gives the relationship between the cut-off frequencies and the centre frequency:

$$\boxed{\frac{f_0}{f_1} = \frac{f_2}{f_0}}$$

This means that when the filter is operating at its centre frequency the applied frequency can vary by the same factor in each direction before reaching the –3dB points.

These filters are much like the filters on a graphic equaliser. Pushing the slider up enables the amplifier to amplify a range of frequencies more than the next.

Example

What is this value of f_0/f_1 and $\frac{f_2}{f_0}$ for the above filter?

Quality factor (Q)

The Q (quality factor) of the filter is a measure of its selectivity. The narrower the bandwidth the higher the Q and the better the selectivity. It is the ratio of the centre frequency to the bandwidth:

$$\boxed{Q = \frac{f_0}{BW}}$$

For the above filter:

$$Q = \frac{1265}{3600}$$

$$= 0.351$$

So, how do you make a passive band-pass filter? The frequency response curve gives you a clue. Look at Figure 2.15.

A high pass filter is joined with a low pass filter. The cut-off frequency of each is such that there is an overlap between the two pass bands. Only those frequencies that lie between the two cut-off frequencies will get through unattenuated. This becomes the pass band for the band pass filter.

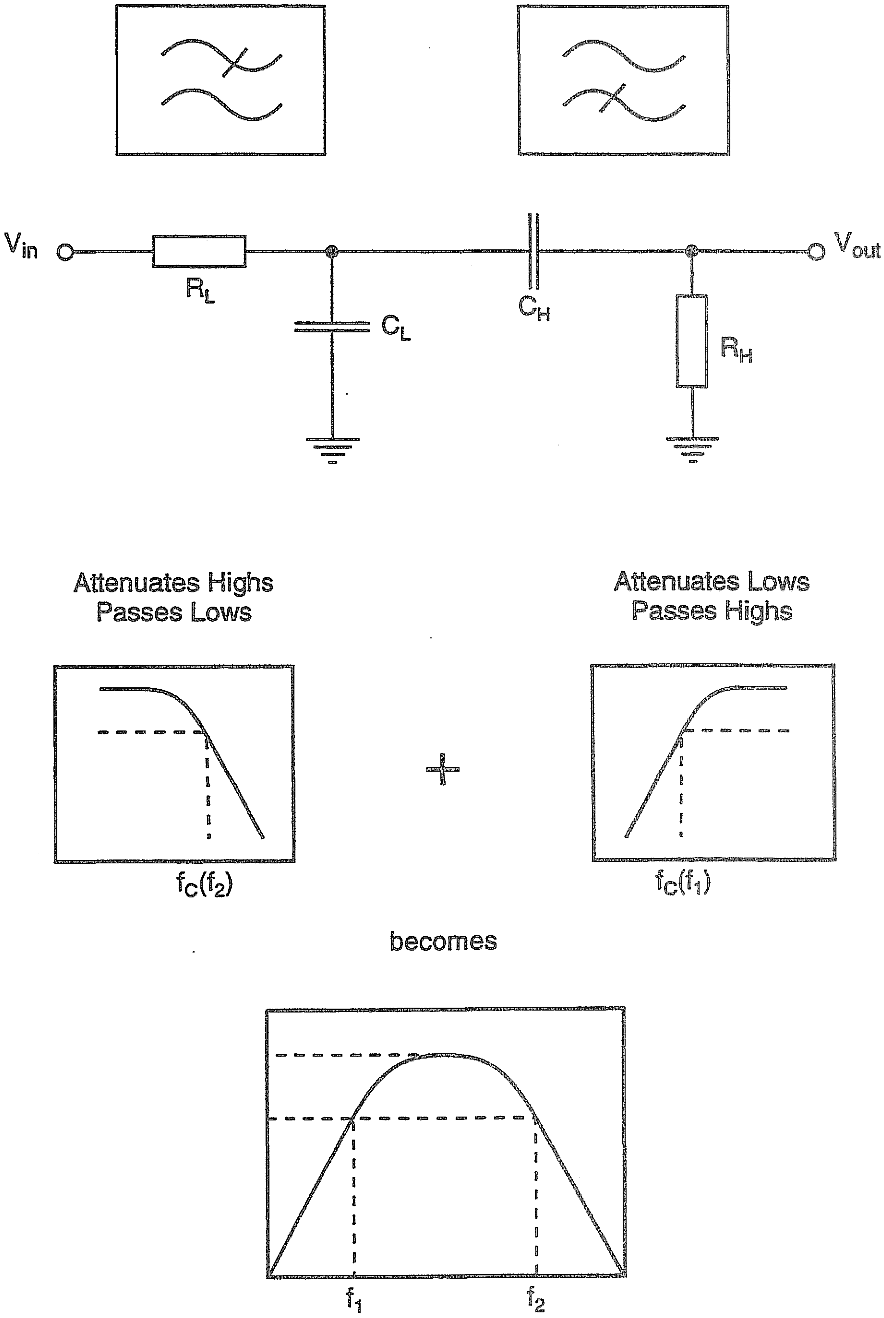


Figure 2.15 Cascading HP and LP filters to form a band pass filter

Cascading passive filters

In the sections you've just studied you saw that filters can be cascaded to either increase the slope in roll off or form bandpass filters.

Cascading passive filters is a little trickier than it seems. You see the second filter forms a load for the first filter. This load varies with frequency. This effect changes the frequency response of the first filter. Similarly the first filter is the source for the second filter. This source has a frequency dependant impedance. Because of that frequency dependence the characteristics of the second filter are affected. To calculate the overall effect requires a strong grasp of circuit theory, so we won't get into that in this module.

You can use everything you've learned in the previous sections as long as you keep the cut off frequencies of the different filters over one decade apart. If you do that, their interactions are negligible and they behave like two separate filters.

Filters using inductors

All the filters we've looked at can also be made using inductors. Remember inductive reactance increases with frequency. So they tend to block high frequencies.

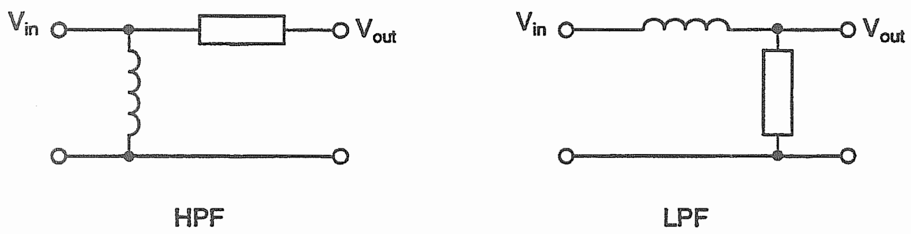


Figure 2.16 First order inductive filters

For the first order filters the cut off frequencies are given by:

$$fc = \cancel{\frac{1}{2\pi RL}} \qquad f_C = \frac{R}{2\pi L}$$

Combining the capacitors and inductors produces higher order filters. Just one capacitor and one inductor produces a second order filter. The roll off will be 40dB per decade.

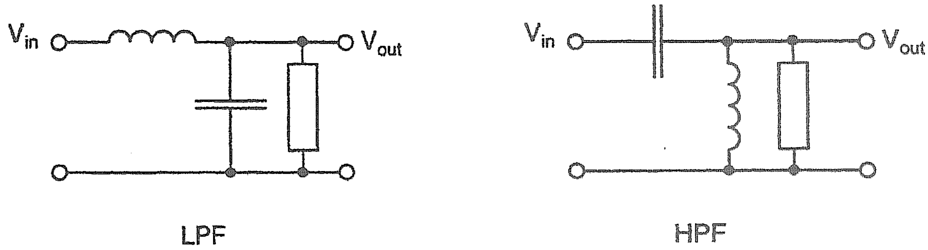


Figure 2.17 Second order L-C filters

Practical

You are ready to do a practical task on filters. Detailed information on Practical Task 2 is located at the end of this chapter.

2.5 Band stop (or notch) filter

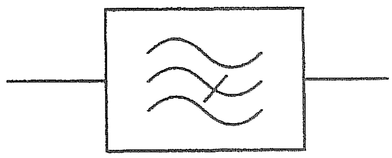


Figure 2.18 Symbol for a band stop filter

As its name suggests (and as the symbol in Figure 2.18 shows) this filter blocks a range or band of frequencies but passes frequencies that are above or below this band. The frequency response curve for a band-stop filter is shown in Figure 2.19.

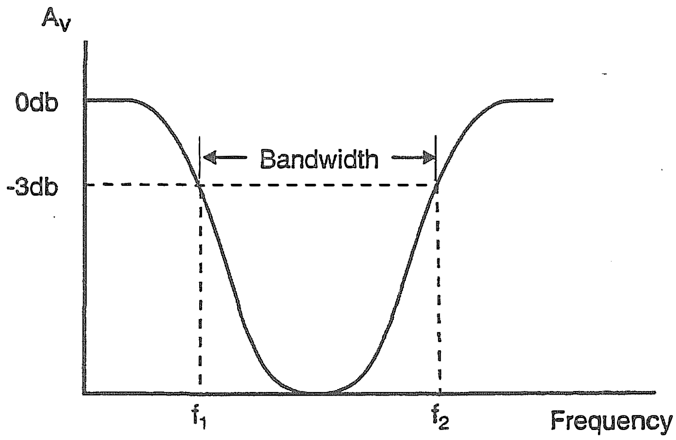


Figure 2.19 Band stop filter frequency response curve

The stop band of this filter is from the lower cut-off frequency, f_1, up to the upper cut-off frequency, f_2.

The bandwidth of this filter is found by:

$$\boxed{BW = f_2 - f_1}$$

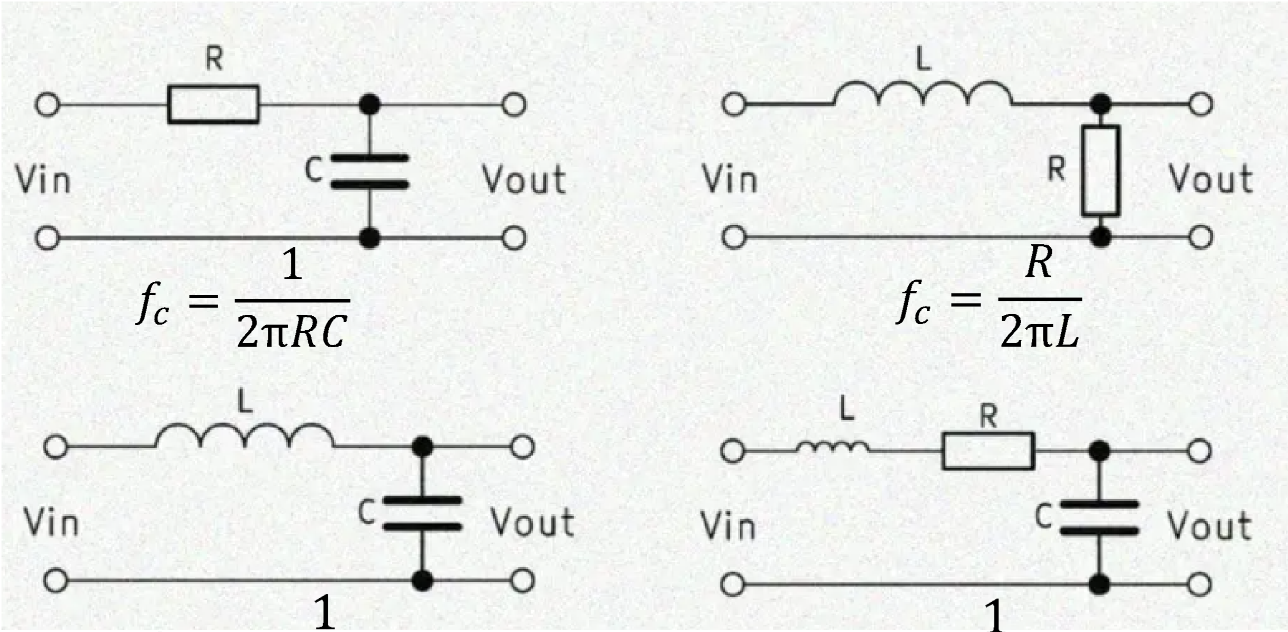
Can you produce a passive band stop filter simply by cascading high and low pass filters?

No! The roll off of the low pass filter would prevent the high pass filter from ever seeing the higher frequencies.

You need to place the two filters in parallel, with their outputs connected to some sort of summing device. This device would then produce an output that was proportional to the sum of its inputs. This can be achieved by using an active filter.

Summary

Low Pass Filters



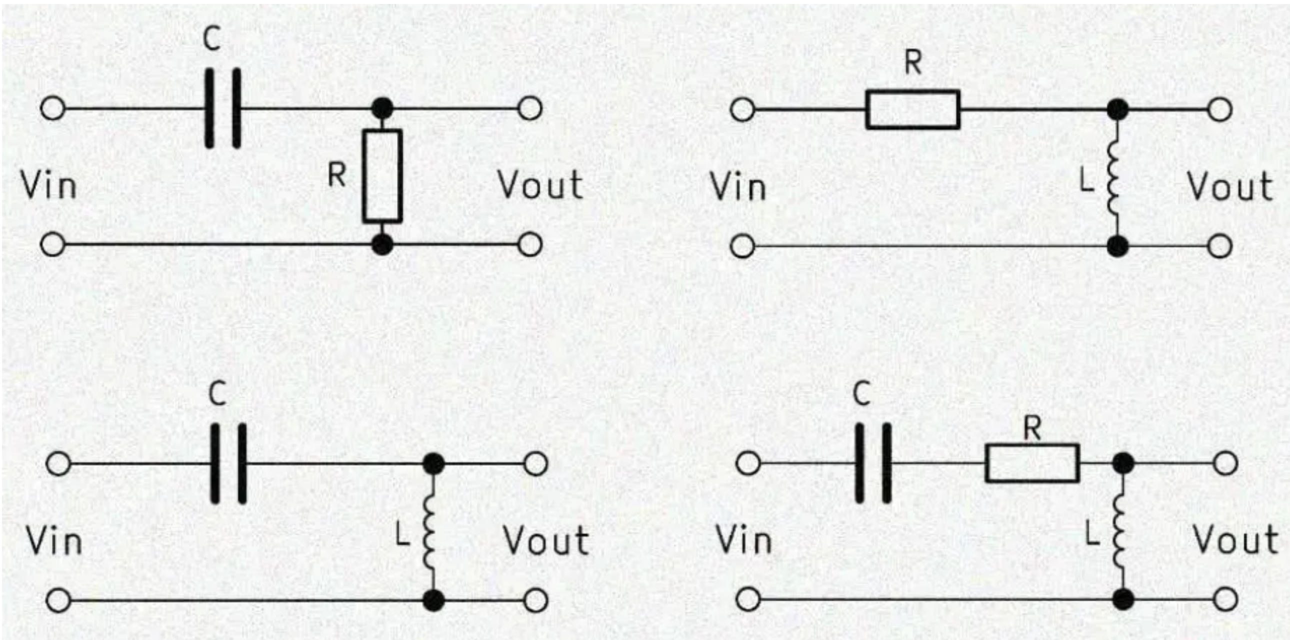
$$f_c = \frac{1}{2\pi RC}$$

$$f_c = \frac{R}{2\pi L}$$

$$f_c = \frac{1}{2\pi\sqrt{LC}}$$

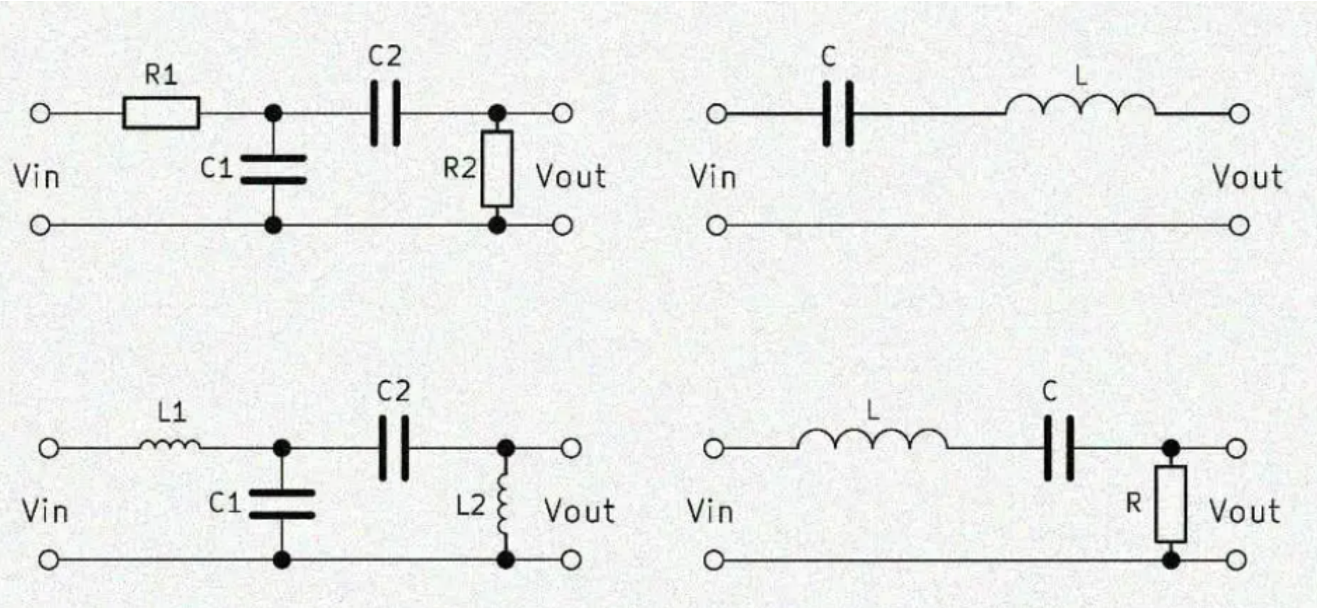
$$f_c = \frac{1}{2\pi\sqrt{LC}}$$

High Pass Filters



https://electronicbase.net/low-pass-filter-calculator/
https://electronicbase.net/high-pass-filter-calculator/

BandPass Filters

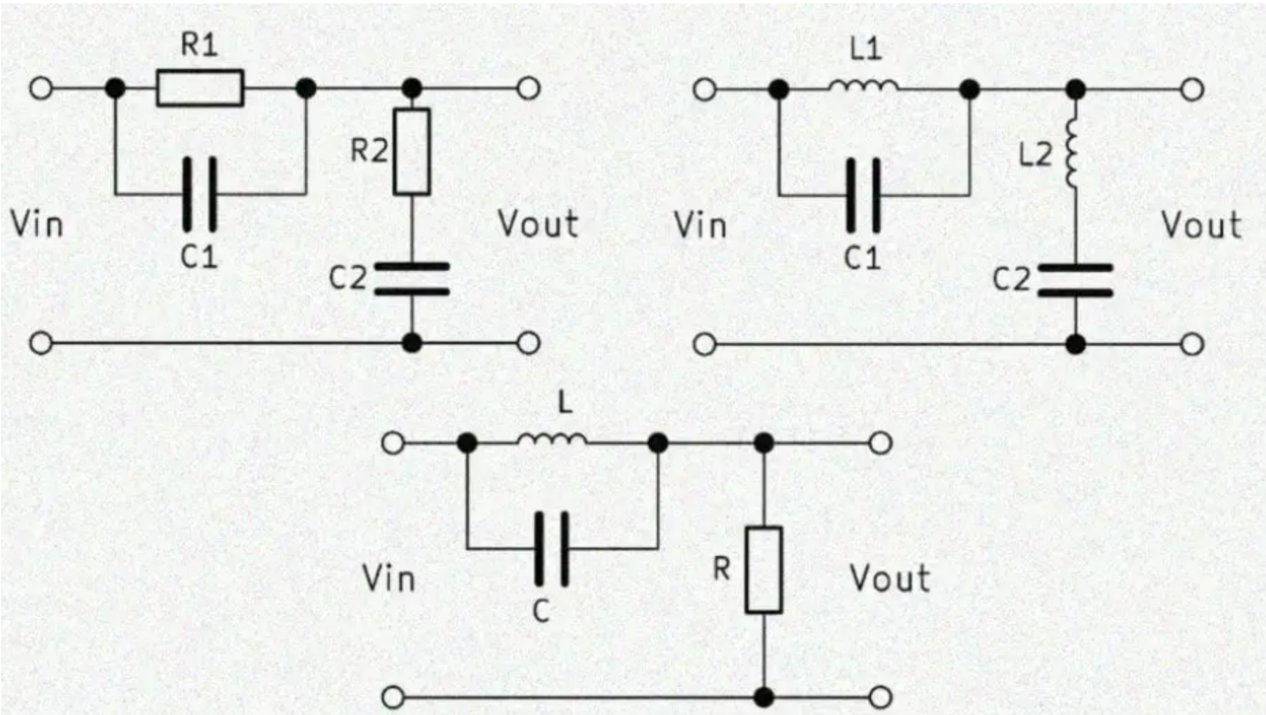


$$f_c = \sqrt{f_H \,\text{x}\, f_L}$$

$$BW = f_H - f_L$$

$$Q = \frac{f_c}{BW}$$

BandStop Filters



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